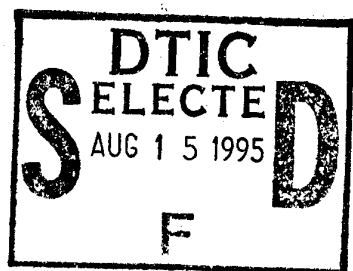


# NONLINEAR PHYSICS THEORY AND EXPERIMENT

INTERDISCIPLINARY WORKSHOP ON NONLINEARITY  
IN PHYSICAL SCIENCES

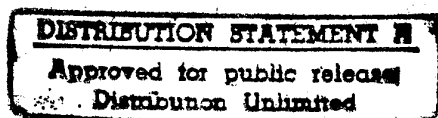
*nature, structure and properties of nonlinear phenomena  
in physics and applied mathematics*

Gallipoli, Lecce (Italy)  
June 29 – July 7 1995



PROCEEDINGS

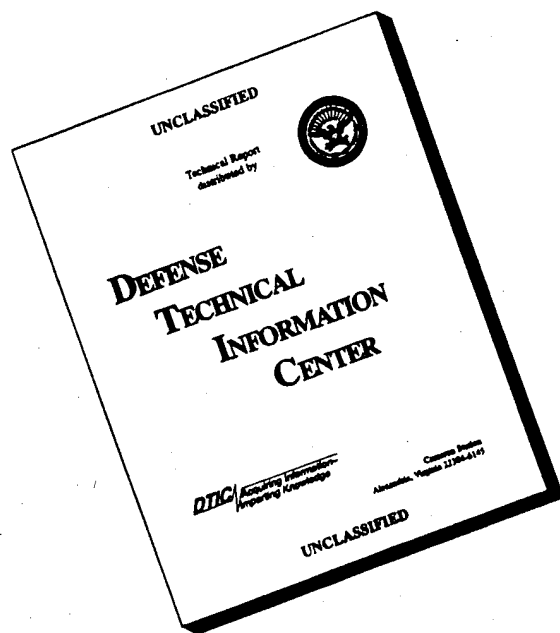
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# 1<sup>st</sup> WORKSHOP ON NONLINEAR PHYSICS THEORY AND EXPERIMENT

Nature, Structure and Properties of Nonlinear Phenomena

in Physics and Applied Mathematics

Le Sirenuse, Gallipoli (Lecce), Italy

June 29 - July 7, 1995

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## 1 Introduction

The Workshop took place from June 29 to July 7 (1995) at the Hotel "Le Sirenuse", a modern building on a sunny sandy beach of the Jonian Sea near Gallipoli, in Southern Italy. There were 93 participants from 22 countries.

The purpose of the Workshop (tentatively the first of a series) was to bring together scientists whose common interest is the nature, structure and properties of nonlinear phenomena in various areas of physics and applied mathematics.

An emphasis was made on both theory and experiments, the underlying objective being to realize a truly interdisciplinary workshop as all these domains have a lot to learn and teach one another.

In fact, topics covered at the Workshop run from nonlinear optics to molecular dynamics, plasma waves, hydrodynamics, quantum electronics and solid

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state, and from inverse scattering transform methods to dynamical systems including integrability, hamiltonian structures, geometrical aspects, and turbulence and chaos.

The proceedings of this meeting will be published by World Scientific (Singapore).

The Workshop was mainly organized by F. Pempinelli and M. Boiti, who profited by the help and advice of J. Léon. We warmly acknowledge the Chairman of the Scientific Advisory Committee, M. Kruskal, and the members of the Committee, M. Barthes, D. Campbell, A. Hasegawa, B. Konopelchenko, A. Osborne, R. Parmentier, P. Sabatier, G. Soliani, K. Spatschek. All logistic and administrative problems have been excellently managed by M. C. Gerardi.

Financial support from the University of Lecce, from Consorzio EINSTEIN (European Institute of Nonlinear Studies via Transnationally Extended Interchanges), from IUPAP (International Union of Pure and Applied Physics), from the INFN (Istituto Nazionale di Fisica Nucleare) and from the University of Rome "La Sapienza" made the conference possible.

We wish also to thank the United States Air Force European Office of Aerospace Research and Development, and the United States Army European Research Office for their contribution to the success of this conference.

In addition to the scheduled program reported below, there were ample opportunities for informal discussions.

The next Workshop of this series will probably be held in the summer of 1997 or 1998. Anybody who wishes to get relevant information should notify the organizers of this Workshop, asking to be put on the mailing list.

## 2 Program

### 29 June

#### *Morning Session*

P Sabatier: "Patchwork approach to nonlinearity, inverse problems, and interdisciplinaryity?" S De Lillo: "NLS solitons under stochastic forcing."

C Rogers, and W K Schief: "Generalized Ermakov systems."

M Grundland: "Conditionally symmetries for nonlinear PDEs."

#### *Afternoon Session*

M A Manna, R A Kraenkel, J C Montero, and J C Pereira: "Long-waves in nonlinear dispersive systems, multiple-time solutions and the KdV hierarchy."

V V Sokolov: "On the Darboux integrable nonlinear hyperbolic equations."

L Bogdanov, and B Konopelchenko: "Lattice and q-difference Darboux-Zakharov-Manakov systems via  $\bar{\partial}$ -dressing method."

L Martina, O K Pashaev, and G Soliani: "Topological field theory and nonlinear  $\sigma$ -models on symmetric spaces."

O K Pashaev: "Integrable Chern-Simons field theory in  $2 + 1$  dimensions."

P G Grinevich, and S P Novikov: "String equation. Physical solution."

### 30 June

#### *Morning Session*

J Leon: "Long-pulse experiments in gas: Spectral Transform and interpretation."

W K Schief, and C Rogers: "The geometry of the LKR system: application of a Laplace transformation to Ernst-type equations" E Doktorov, and V Shch-esnovich: "Nonlinear initial-boundary evolutions with singular dispersion laws associated to the quadratic bundle."

F Pempinelli, M Boiti, and J Leon: "Nonlinear discrete systems, singular dispersion relations and Bäcklund transformations."

P Caudrey: "Some thoughts on integrating non-integrable systems."

#### *Afternoon Session*

M Musette: "Non-fuchsian extension to the Painlevé test."

R Conte: "Beyond the two-singular manifold method."

A Pickering: "Lax pairs and exact solutions from Painlevé analysis."

G Tondo: "On the integrability of Henon-Heiles type systems."

V S Dryuma: "Geometric method in theory of nonlinear dynamical systems with regular and chaotic behaviour."

### 1 July

#### *Morning Session*

V K Mel'nikov : "On the existence of self-similar structures in the resonance region."

B Malomed, R Grimshaw, and J He: "Propagation of a soliton in a periodically modulated nonlinear waveguide."

E Celeghini: "Statistics of particles and Quantum Groups."

K H Spatschek, E W Laedke , and O Kluth : "On the existence and stability of solitary wave solutions in discrete nonlinear Schrödinger systems."

R Willox: "Symmetry constraints of the KP hierarchy and a nonlocal Boussinesq equation."

#### *Afternoon Session*

A M Kosevich, A S Kovalev, and I M Babich: "Solitons and localized excitations in a 1D chain with the long-range interaction."

A G Shagalov: "Defect-like solutions of 2D Sine-Gordon equation as a model of inhomogeneous states in large area Josephson junctions."

R Yamilov, I Yu Cherdantsev, and S I Svinolupov : "Local master symmetries of nonlinear integrable evolution equations."

S I Svinolupov: "Multicomponent integrable equations, non associative algebras and affinely connected spaces."

*Theatre, Mimo and Dance (ARACNE Mediterranean Group)*

## 2 July

### *Morning Session*

P Winternitz: "Symmetries of differential difference equations."

D Levi: "Lie symmetries for differential difference equations."

M Boiti, F Pempinelli, and A Pogrebkov: "Solving the Kadomtsev-Petviashvili equation by using an extension of the resolvent theory."

A Pogrebkov, M Boiti, and F Pempinelli: "Problems of the two-dimensional scattering on a non trivial background."

R Carroll, and Y Kodama: "Solution of the dispersionless Hirota equations."

### *Afternoon Session*

W X Ma: "Symmetry constraints of soliton equations: binary nonlinearization method."

J G Pereira, R A Kraenkel, and M A Manna: "The role of the Korteweg-de Vries hierarchy in obtaining secular-free perturbative series."

J H Lee: "Solutions of some soliton equations via wavelet basis."

B Gürel, M Gurses, and I Habibullin: "Integrable boundary conditions for evolutionary type equations."

V Dubrovsky: "The application of  $\bar{\partial}$ -dressing method for some 2+1-dimensional nonlinear integrable equations."

### *Movie and Poster Session*

L Martina, and D Perrone: "Movie on multidimensional localized solitons."

M D Cunha, V V Konotop, and L Vazquez: "Small amplitude breathers in a non-local sine-Gordon model."

P P Del Santo, M Scalerandi, and S Biancotto: "Two phase behaviour of porosity and surface width in growth phenomena."

V S Gerdjikov, and A N Kostov: "IST analysis of Stokes-anti-Stokes stimulated Raman scattering."

K Imai, and K Konno: "Discrete space and time equation as a difference scheme for numerical simulation of the corresponding continuous integrable equation."

D Lucchetta: "Anomalous scaling and sign-singular measures in electro-convective turbulence."

E M Maslov, and A G Shagalov: "On dynamics of pulson collisions."

Mercaldo, C Attanasio, C Coccorese, and L Maritato: "Superconducting and spin glass interplay in coupled layered structures."

M Palese, E Alfinito, M Leo, R A Leo, and G Soliani: "Algebraic and geometrical properties of nonlinear integrable evolution equations."

## 3 July

### *Excursion to Otranto and Santa Cesarea*

### *Afternoon Session*

B Konopelchenko: "Generalized Weierstrass inducing: constant mean curvature surfaces via integrable dynamical systems."

W Oevel: "Darboux transformations for integrable lattice systems."  
 N A Slavnov: "Multipoint correlation functions in one-dimensional impenetrable Bose-gas."  
 V E Adler: "On the rational solutions of the Shabat equation."  
 Y M Berezansky: "Nonispectral flows on Semi-infinite Jacobi matrices."

#### 4 July

##### *Morning Session*

A Hasegawa: "Recent progress of applications of optical solitons for ultra-fast communication."  
 S Venakides: "1) The Toda shock problem (with P Deift and R Oba). 2) Forced lattice vibrations (with P Deift and T Kriecherbauer)."  
 V Tognetti, C Biagini, A Cuccoli, P Verrucchi, and R Vaia: "The quantum easy-plane ferro- and antiferromagnet."  
 M Remoissenet: "Dynamics of blood pressure waves in large arteries."

##### *Afternoon Session*

I Barashenkov, M M Bogdan, and M Bondila: "Topography of attractors of the parametrically driven nonlinear Schrödinger equation."  
 V Gerdjikov, I O Uzunov, M Göles, and F Lederer: "N-soliton interactions. A generalization of the Karpman-Soloviev approach."  
 T A Minelli: "1) Wavelet analysis of the electrocortical activity (with L Battiston, C Gabrielli and A Pascolini). 2) Nonlinear simulation of the electrocortical activity (with L Turicchia)."  
 A Nepomnyashchy, and D E Bar: "Nonlinear waves generated by instabilities in presence of a conservation law."

#### 5 July

##### *Morning Session*

D Campbell: "Semiconductor superlattices: from order to chaos."  
 R Goldstein: "The geometry of nonlinear dynamics from Turing patterns to superconductors."  
 S Flach: "Existence and properties of discrete breathers."  
 K Konno, and H Kakuwata: "Inverse problem, linearization and related topics of coupled integrable dispersionless equation."  
 A Parker: "From solitary waves to periodic waves via nonlinear superposition: a bilinear approach."

##### *Afternoon Session*

A V Lazuta: "Second harmonic of nonlinear response of magnets."  
 A I Visinescu: "Thermodynamic functions of nonlinear 1-D systems."  
 A Vinogradov: "New geometrical methods of analysis of nonlinear PDE."  
 H Oono: "N-soliton solution of Harry-Dym equation by Inverse Scattering method."

## *Social Dinner*

### **6 July**

#### *Morning Session*

M Barthes, and G De Nunzio: "Proton dynamics along the hydrogen bond in chains of peptide groups: polarons or proton transfer?" J Smith: "Dynamics of molecular crystals: simulations versus experiment."

I L Bogolubsky, and A A Bogolubskaya: "1) String-like solitons in gauged models of anisotropic Heisenberg antiferromagnet (Soliton analogs of Abrikosov-Nielsen-Olesen vortices (strings)). 2) Two-component localized solutions in a nonlinear DNA model."

R Parmentier: "Coupled Josephson arrays."

A S Kovalev: "Compactons and picons in magnetically ordered media and relation between them and interacting Bose-gas problem."

*Visit to Lecce*

### **7 July**

#### *Morning Session*

A Osborne: "Inverse scattering transform in the theta function representation. Theory, numerical methods and data analysis."

D Müller, A F Shchepkin, and J J O'Brien: "NLS-solitons in covariant shallow water."

S Leibovich: "The upper ocean: symmetry breaking, coherent structures, and mixing."

P Santini: "1) Multiscale expansions in physics and the NLS hierarchy (with A Degasperis, S. Manakov). 2) Solitons, compactons and an inverse acoustic problem (with A. Fokas)."

O Mokhov, and E Ferapontov: "Equations of associativity and systems of hydrodynamic type."

#### *Afternoon Session*

A Khrennikov: "p-adic description of chaos."

T I Lakoba, and D J Kaup: "How the variational method gives rise to false instabilities for 1-D solitary waves."

V Rosenhaus, and G H Katzin: "On conservation laws and symmetries for nonlinear differential equations."

M Kruskal: "Extensions and variants of the Painlevé test, exponential asymptotics, and the eighth asymptotological principle."

*Discussion on Future Workshops and Closing Ceremony*



12/07/95

NONLINEAR PHYSICS. THEORY AND EXPERIMENT  
Gallipoli, 29 Giugno - 7 Luglio 1995

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NONLINEAR PHYSICS. THEORY AND EXPERIMENT  
Gallipoli, 29 Giugno - 7 Luglio 1995

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## On the rational solutions of the Shabat equation

V.E. Adler

Recently A.B. Shabat introduced a new exactly solvable example of the Sturm-Liouville operator  $L = -D^2 + u$  with spectrum which forms infinite geometric progression  $\lambda_j = -q^{2j}$ ,  $j = 0, 1, 2, \dots$ ,  $0 < q < 1$ . The potential  $u = 2v'$  is defined in terms of the ordinary differential equation with deviating argument, which we write in the form

$$q^2 v'(qx) + v'(x) = (qv(qx) - v(x))^2 - 1. \quad ((1))$$

This equation describes self-similar reduction of the so-called dressing chain, that is sequence of Darboux transformation for the Sturm-Liouville operators. Till now some papers devoted to formal algebra underlying the Shabat approach were published, but very few information about analytical properties of the eq. (1) solutions were obtained. In this lecture I should like to present some results in this direction.

The main observation is that eq. (1) admits, in turn, the Darboux transformation which generates discrete symmetry group of this equation. This allows to prove some results about analytical nature of the function  $v$ . For example, we prove that  $v$  is meromorphic in the whole complex plane. Moreover, the Darboux transformation makes possible to construct infinite sequence of the exact rational solutions. This solutions can be found by iterative procedure suggested by M. Adler and Moser for constructing of the KdV rational solutions. In fact, our solutions are rational solutions of the KdV with suppressed dependence on  $t$  and uniquely fixed integration constants.

# Topography of Attractors of the Parametrically Driven Nonlinear Schrödinger Equation

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The parametrically driven, damped NLS equation is numerically simulated in the neighborhood of its exact soliton solution. We obtain the attractor chart on the control parameter plane in the domain of the soliton instability. Regions of the period-doubling and quasiperiodic transitions to chaos are found, and the existence of a critical point where the two scenarios meet, is demonstrated.

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# PROTON DYNAMICS ALONG THE HYDROGEN BOND IN CHAINS OF PEPTIDE GROUPS : POLARONS OR PROTON TRANSFER ?

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## Abstract :

The proton dynamics is compared in two crystalline chains of (...OCNH...) peptide units having neighboring structures (acetanilide ACN , and N-methylacetamide NMA) .

The proton of the  $>\text{N}-\text{H}\cdots\text{O}=\text{}$  bond in ACN is submitted to a simple-well potential , as demonstrated by the neutron diffraction structural determination , whereas tautomerism is suspected in NMA (  $>\text{N}-\text{H}\cdots\text{O}=\text{}$  , and  $>\text{N}\cdots\text{H}-\text{O}-$  ).

In both systems some vibrations of the peptide group atoms display non harmonic or anomalous properties . In ACN the observed extra-intensities are assigned to nonlinear coherent excitations ( polarons ) , whereas a proton transfer model is proposed for NMA, arising from dynamic exchange between the two resonance forms .

A new examination of the temperature dependence of the amide modes in the infrared spectra shows very close behaviors in both compounds, and does not support the recent proposal of a NH stretching vibration at  $\sim 1595\text{ cm}^{-1}$  in NMA , resulting from the hypothesis of proton transfer.

Amide vibrations in other peptide chains will be also discussed.

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Yuriy M. Berezansky (Kiev, Ukraine; Lublin, Poland)

Nonisospectral Flows on Semi-infinite Jacobi Matrices

It is proved that if the spectrum and spectral measure of a semi-infinite Jacobi matrix  $L(t)$  change appropriately, then  $L(t)$  satisfies a generalized Lax equation of the form  $\dot{L}(t) = \Phi(L(t), t) + [L(t), A(L(t), t)]$  where  $\Phi(\lambda, t)$  is a polynomial with  $t$ -dependent coefficients and  $A(L(t), t)$  is a skew-symmetric matrix which is determined by the evolution of the spectral data. Such an equation is equivalent to a wide class of generalized Toda lattices. The theory of Jacobi matrices gives rise to the procedure of solution of the corresponding Cauchy problem by the inverse spectral problem method. The linearization of this nonlinear equation in terms of the moments is established.

# Lattice and q-difference Darboux-Zakharov-Manakov systems via $\bar{\partial}$ -dressing method

L.V. Bogdanov\* and B.G. Konopelchenko  
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## Abstract

A general scheme is proposed for introduction of lattice and q-difference variables to integrable hierarchies in frame of  $\bar{\partial}$ -dressing method. Using this scheme, lattice and q-difference Darboux-Zakharov-Manakov systems of equations are derived. Darboux, Bäcklund and Combescure transformations and exact solutions for these systems are studied.

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## Two-component localized solutions in a nonlinear DNA model

A.A.Bogolubskaya, I.L.Bogolubsky

One of the most important and interesting problems of contemporary molecular biophysics is to find mechanisms of excitation transmission along the macromolecular chains of proteins and nucleic acids. We consider a new nonlinear DNA model which can be named "a nonlinear analog of the Volkov-Kosevich DNA model" (NVKM). On the other hand, our model is an extension of one/two-component models describing nonlinear torsional DNA base motions (Englander et al, Yomosa, Homma and Takeno, Yakushevich et al) since our model describes interaction of torsional base displacements of sugar-phosphate "backbone" rods. Within NVKM torsional motions are governed by the potential  $V=(1-\cos u)$ , and torsional and transverse displacements in DNA double helix interact through hydrogen bonds, which are described in harmonical approximation. As a result we obtain a nonlinear set of 4 PDEs describing evolution of initial excitations (two Eqs. for each of two DNA rods). By pointing out zero solution for DNA motions as a unit (zero bending and torsion), we reduce the 4-component NVKM to the 2-component NVKM. The latter model describes "pure" macromolecule "breathing", i.e. relative internal motions within DNA double helix.

Soliton and breather solutions are found within 2-component model. We consider breathers as effective energy and information carriers along macromolecule chains.

For a special choice of values of parameters our model possesses 2-component N-soliton solutions.

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Title of the poster:

String-like solitons in gauged models of anisotropic Heisenberg antiferromagnet

I.L.Bogolubsky, A.A.Bogolubskaya

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Igor L. Bogolubsky

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Scientific interests: D-dimensional nonlinear systems.

- 1974...Solitons and collapse in strong Langmuir turbulence, described by Zakharov's set of Eqs.
- 1975...Interaction of solitons in regularized (in Cauchy sense) Boussinesq equation
- 1976...Evolution of spherically/axi-symmetric "domain walls" within nonlinear scalar field models with degenerated vacua, formation of localized oscillating high-amplitude solutions ("pulsons")
- 1979... Nonlinear integro-differential spectral problem for nonlocal Schrodinger equation arising in Witten's nonrelativistic potential model (and its generalizations comprising linearly rising potentials) of baryons in  $1/N$ -expansion of QCD (quantum chromodynamics)
- 1981...Lattice quantum field theory
- 1983...Topological solitons and their bound states within nonlinear N-component sigma-models in D-dimensional space (unit isovector models) of (hopefully) universal (condensed matter, particle and nuclear physics, cosmology) importance.
- 1991...multicomponent nonlinear DNA and protein models.
- 1993...integrable chiral field models of physical importance

# Solving the Kadomtsev–Petviashvili equation by using an extension of the resolvent theory

M. Boiti, F. Pempinelli and A. Pogrebkov

Dipartimento di Fisica – Università di Lecce – Lecce, Italy

The Kadomtsev-Petviashvili I (KPI) equation is considered as a useful laboratory for experimenting with new theoretical tools able to handle the specific features of integrable models in  $2+1$  dimensions. It is proposed an extension of the concept of resolvent. This extended resolvent defined for the nonstationary Schrödinger equation, which is the spectral equation related to KPI, results to be the only mathematical entity needed in defining Jost solution and Spectral Data. Analytical properties of Jost solutions and characterization equations of Spectral Data are derived by using the Hilbert identities satisfied by the resolvent.

This approach allows one to study in details the singular behavior of solutions of the KPI equations at  $t = 0$  in the case of unconstrained initial data and to consider the case in which the solution is going to a constant along a finite number of directions at large distances.

# SEMICONDUCTOR SUPERLATTICES: FROM ORDER TO CHAOS

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Modern molecular epitaxy techniques have made possible the fabrication of "superlattices" consisting of alternating nanometer-scale layers of different semiconducting materials, such as  $GaAs$  and  $Ga_xAl_{1-x}As$ . For electrons moving perpendicular to the planes of these layers, these "semiconductor superlattices" (SSL) appear to be "artificial" one-dimensional solids, since the layers create equally spaced periodic potential wells. Instead of being determined by the natural bond lengths of the atoms in an ordinary solid, the spacing of these potential wells in the superlattices is an easily varied parameter and can be chosen to amplify a number of physical effects that are masked or unobservable in normal solids. Importantly, since many of these effects are associated with the response to applied electric fields, they can have potential applications in technological devices.

In this colloquium level presentation, after introducing the basic concepts and fabrication methods of SSL, we describe a number of interesting phenomena that occur in the presence of static electric fields, including negative differential resistance, Bloch oscillations, and the "Wannier-Stark ladder".

Turning to the case of time-dependent fields, we show that still more complex phenomena can occur. Using a balance-equation approach, we show that proper accounting for the collective excitations of the superlattice electrons can lead to complicated dynamics, including both transient and stationary deterministic chaos. We estimate the conditions for experimental observation of this deterministic chaos and discuss the similarities of the nonlinear dynamics in the superlattice-field system with those in bistable optical devices and lasers.

# **TURBULENCE IN THE SOLAR WIND: ANOMALOUS SCALING LAWS AND MULTIFRACTALS**

by

Vincenzo Carbone<sup>(1)</sup>, Roberto Bruno<sup>(2)</sup>, and Pierluigi Veltri<sup>(1)</sup>

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We present the investigations of the large-amplitude fluctuations of the velocity field in the solar wind magnetohydrodynamic (MHD) turbulence. We use some data sets coming from satellite observations in the inner solar wind. We calculate the scaling exponents of the  $q$ -th order velocity structure functions, showing the presence of intermittency evidenced through anomalous scaling laws. We show that these scaling exponents are in excellent agreement with the multifractal model describing the nonlinear energy cascade in magnetohydrodynamic.

# SOLUTION OF THE DISPERSIONLESS HIROTA EQUATIONS

R. Carroll\* and Y. Kodama†

May, 1995

## Abstract

The dispersionless differential Fay identity is shown to be equivalent to a kernel expansion providing a universal algebraic characterization and solution of the dispersionless Hirota equations. Some calculations based on  $\bar{D}$ -bar data of the action are also indicated.

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## STRUCTURE(S) OF THE FOCK SPACE

E. CELEGHINI

We discuss the representations of the second quantization algebras and their products, by means of the Hopf coalgebra.

Because the fact that particles cannot be distinguished is a general property of the Fock space and not of a peculiar algebra, the different physical properties are related to many-particles systems and to the first quantization levels.

Because of the cocommutativity,  $h(1)$  is shown to prescribe the complete equivalence of levels, that can be broken introducing the  $q$ -deformation.

In particular, considering two levels only,  $h(1)$  gives us the binomial distribution with  $p=1/2$ , while  $h_q(1)$  allows us to obtain, in function of  $q$ , all the values of  $p$ .

A completely different statistics' (perfectly compatible with the axioms) is obtained from the superalgebra  $osp(1|2)$  and its  $q$ -deformation  $osp_q(1|2)$ .

## THE TWO-SINGULAR MANIFOLD METHOD

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**Abstract** – For its coherence, the theory of singularity analysis must be able to provide the auto-Bäcklund transformation of a partial differential equation (PDE) when there is one. The singular manifold method of Weiss [6] does that, but it only applies to PDEs possessing one family of movable singularities.

We have developed a "two-singular manifold method" [5, 2] for PDEs with two such families with opposite principal parts, which provides the auto-BT of sine-Gordon, modified KdV and the classical Boussinesq system or Broer-Kaup system [1, 3]

$$\left. \begin{aligned} U_t + \left( V + \frac{1}{2} U^2 \right)_x &= 0 \\ V_t + (a^2 U_{xx} + UV)_x &= 0. \end{aligned} \right\} \quad (1)$$

In this last case for instance, the scalar equation for one the two components, namely  $\int^\infty U dx$ , has the two-family feature, allowing to derive the Lax pair and the Darboux transformation — and hence the auto-Bäcklund transformation [4] — for the classical Boussinesq system from its Painlevé analysis only.

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S. De Lillo

## NLS solitons under stochastic forcing

### Abstract

The effects of gaussian noise on NLS solitons are analyzed in the case of external noise of the multiplicative type, suitable to model local inhomogeneities effects in an optical fiber. The two cases of weakly correlated and of strongly correlated noise are considered. The statistical properties of the system and the relevant two-point correlation functions are obtained and compared for the two types of gaussian noise.



# Nonlinear Evolutions with Singular Dispersion Laws Associated to the Quadratic Bundle

E. Doktorov and V. Shchesnovich  
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A class of nonlinear equations with the singular dispersion relation associated with the AKNS problem had been exhaustively studied by J. Leon (1990). It contains, in particular, the equations describing a propagation of picosecond optical solitons in media with both resonant (McCall-Hahn-type) and nonresonant (Kerr-type) nonlinearities (E. Doktorov and R. Vlasov 1982). Below we generalize these results to the case of the quadratic bundle.

The  $\bar{\partial}$ -problem  $\bar{\partial}\psi(k) = \psi(k)R(k)$  is accompanied with  $x$ - and  $t$ -dependences of the spectral transform  $R$ :  $R_x = (i/\alpha)(k^2 + \beta)[R, \sigma_3]$ ,  $R_t = (\omega_r + \omega_s)[R, \sigma_3]$ , where regular  $\omega_r$  and singular  $\omega_s$  parts of the dispersion law are taken in the form

$$\omega_r(k) = \sum_{j=0}^J \gamma_{2j} k^{2j}, \quad \gamma_{2j} = \text{const}, \quad \omega_s(k) = \frac{1}{2\pi i} \iint \frac{dl \wedge d\bar{l}}{l^2 - k^2} l^2 \rho(l^2), \quad \bar{\partial}\omega_s = k\rho(k^2).$$

A hierarchy of integrable systems of equations is

$$Q_t = -2\sigma_3 \sum_{j=0}^J \gamma_{2j} \Lambda^j Q + \frac{2i}{\alpha} \sigma_3 \langle k\rho(k^2)M(k) \rangle, \quad (1a)$$

$$\langle \rho(k^2) \left( M_x + \left[ \frac{i}{\alpha} (k^2 + \beta + \frac{1}{2} \alpha^2 Q^2) \sigma_3 - kQ, M \right] \right) \rangle = 0. \quad (1b)$$

Here  $M = \psi \sigma_3 \psi^{-1}$ ,  $\langle f \rangle = (2\pi i)^{-1} \iint dk \wedge d\bar{k} f(k)$ ,  $\rho(k^2)$  is an arbitrary function (distribution),  $\Lambda$  is the recursion operator. The gauge transformation  $\psi = g(x, t)\psi'$  leads to a hierarchy of equations among which there is "a modified nonlinear Schrödinger equation with a source":

$$iE_t + E_{xx} - i\alpha(|E|^2 E)_x + 2\beta|E|^2 E = (2i/\alpha)p + (2/\gamma)n,$$

$$p_x + 2i\nu p = 2\gamma En, \quad n_x = \gamma(E\bar{p} - \bar{E}p), \quad \nu = (\gamma^2 + \beta)/\alpha, \quad \gamma = \text{const}. \quad (2)$$

These equations describe a propagation of femtosecond optical pulses in nonlinear waveguides with resonant impurity atoms. Soliton solution of (2) is obtained.

# GEOMETRIC METHOD IN THEORY OF NONLINEAR DYNAMICAL SYSTEMS WITH REGULAR AND CHAOTICAL BEHAVIOUR

V.S.DRYUMA (MOLDOVA)

A geometric method for investigation of the nonlinear dynamical system in the form

$$\begin{aligned} dx/dt &= P(x, y, z, A_i), \\ dy/dt &= Q(x, y, z, A_i), \\ dz/dt &= R(x, y, z, A_i), \end{aligned} \quad (1)$$

with parameters  $A_i$  is given. We use here the equivalence between the system (1) and corresponding second order ordinary differential equation:

$$y'' = f(x, y, y', A_i) \quad (2)$$

in the space with coordinates  $(x, y)$ , where  $(y' = dy/dx)$ .

The properties of the equation (2) by its general integral

$$F(x, y, a, b, A_i) = 0 \quad (3)$$

are determined.

In the case of equations (2) of invarianten type:

$$y'' = A(x, y) (y')^3 + B(x, y) (y')^2 + C(x, y) y' + D(x, y) \quad (4)$$

dual equation  $b'' = g(a, b, b' = c)$  with function  $g(a, b, b')$  in form

$$\begin{aligned} &g/aacc + 2cg/abcc + 2gg/accc + c^2 g/bbcc + 2cgg/bccc + g^2 g/cccc + \\ &(g/a + cg/b)g/ccc - 4g/abc - 4cg/bbc - 3gg/bcc - cg/c(g/bcc) - \\ &g/c(g/acc) + 4g/c(g/bc) - 3g/b(g/cc) + 6g/bb = 0 \quad (g/c = dg/dc \dots) \end{aligned}$$

is solved and the properties of general integral (3) for the equations (4) are investigated.

Using the equivalence between geometry of equations (4) under arbitrary change of the coordinates  $x, y$  and the surfaces in projective spaces  $RP$   $\dim=3, 4$  the Hirota-Satsuma equation:  $U/xyy = (U/x)(U/y)$  for the study of the properties of the equations (4) are considered.

The main results of the present geometric approach are following.

The relations between Invariantes of equation (4) are dependent on parameters of the system (1) and have the form  $J(n+1) = \Phi[J_n, J(n-1), A_i]$ , which logistical map  $J(n+1) = A J_n [1 - J_n]$  are generalized.

Projective connection in the set of linear elements  $(x, y, y')$  introducing by the equation (4) depends on parameters of the system (1) and generates the Finsler metrics of various type in each part of the space.

The regular or chaotical behaviour of initial dynamical system (1) is determined by the relations between Invariantes and properties of the Finsler metrics, which are constructed from the equation (4).

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**Isoparametric hypersurfaces in spheres,  
integrable nondiagonalizable systems of hydrodynamic type,  
and  $N$ -wave systems.**

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*Abstract.* Isoparametric hypersurface  $M^n \subset S^{n+1}$  can be defined as the intersection of the unit sphere  $r^2 = u^1 + \dots + u^{n+2} = 1$  with the level set  $F(u) = \text{const}$  of a homogeneous polynomial  $F$  of degree  $g$ , satisfying Cartan-Munzner equations

$$(\nabla F)^2 = g^2 r^{2g-2},$$

$$\Delta F = cr^{g-2},$$

$c = \text{const}$ . We introduce Hamiltonian system of hydrodynamic type

$$u_t^i = \frac{1}{g} \delta^{ij} \frac{d}{dx} \frac{\partial F}{\partial u^j},$$

with the Hamiltonian operator  $\delta^{ij} \frac{d}{dx}$  and the Hamiltonian density  $\frac{1}{g} F(u)$ . Under the additional assumption of the homogeneity of the hypersurface  $M^n$  the restriction of this system to  $M^n$  proves to be nondiagonalizable, but integrable and can be transformed to appropriate integrable reduction of the  $N$ -wave system. Possible generalizations to isoparametric submanifolds (finite or infinite dimensional) are also briefly indicated.

Member of the Scientific committee: Boris Konopelchenko

Biographical presentation:

Ferapontov Eugene V.

"The application of Di-bar dressing method to some 2+1-dimensional integrable nonlinear equations"

V.G.Dubrovsky

Some 2+1-dimensional integrable generalizations of dispersive long wave and nonlinear Schrodinger (or heat) equations are considered. The broad classes of exact solutions of these equations are constructed via Di-bar dressing method.

# ***N*-soliton Interactions. A Generalization of the Karpman-Soloviev Approach.**

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Karpman and Soloviev [1] proposed a method to analyze the behavior of the two-soliton systems of the nonlinear Schrödinger equation under external perturbations, provided the solitons are well separated. Recently it was shown [2, 3, 4], that their results adequately describe the soliton interaction for a number of perturbations, such as bandwidth limited amplification, third order dispersion etc.

In the present paper we derive the set of equations, generalizing the results of [1] to *N*-soliton systems of slightly overlapping solitons. We prove, that in such systems only the nearest neighbours interact, as it was expected [5, 6]. However, the behavior of the inner solitons is qualitatively different from the one of the two end solitons. We discuss the relation between our system and the Toda lattice model and illustrate our results with several numerical simulations.

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# EXISTENCE AND PROPERTIES OF DISCRETE BREATHERS

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Nonlinear classical Hamiltonian lattices exhibit generic solutions in the form of discrete breathers. These solutions are time-periodic and (at least) exponentially localized in space. The lattices exhibit discrete translational symmetry. Discrete breathers are not confined to certain lattice dimensions. Necessary ingredients for their occurrence are the existence of upper bounds on the linear spectrum (of small fluctuations around the groundstate) of the system as well as the nonlinearity. I will present existence proofs, formulate necessary existence conditions, and discuss structural stability of discrete breathers. The following results will be also discussed: the birth of breathers through tangential bifurcation of zone boundary plane waves; dynamical stability; details of the spatial decay; numerical methods of obtaining breathers; interaction of breathers with phonons and electrons; applications.

## THE GEOMETRY OF NONLINEAR DYNAMICS FROM TURING PATTERNS TO SUPERCONDUCTORS

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Many problems in physics and biology involve the conformations and dynamics of filaments, interfaces, and surfaces, from the writhing of supercoiled DNA and the fluctuations of elastic membranes to the fingering of flux domains in superconductors. Associated with these are geometrical and topological constraints on the motion that are central to the physics. This talk will focus on theoretical and experimental work on the dynamics of pattern formation in a class of such systems with long-range interactions: Langmuir monolayers, Type-I superconductors, magnetic films, and chemical reaction-diffusion systems. All exhibit two-dimensional "labyrinthine" pattern formation consisting of an intricately folded interface bounding two coexisting phases. A unifying theoretical framework is developed to treat the nonlinear dynamics of those interfaces, and tested against experiment. These observations are placed in the broader context of nonequilibrium systems and nonlinear dynamics, with emphasis on connections between the differential geometry of curve motion and the dynamics of integrable soliton systems of the KdV and NLS type.

## String equation-2. Physical solution.

P.G.Grinevich, S.P.Novikov.

This paper is a continuation of the paper by S.P.Novikov in *Funct. Anal. Appl.*, v.24(1990), No 4, pp 196-206.

String equation is by definition the equation  $[L, A] = 1$  for the coefficients of two linear ordinary differential operators  $L$  and  $A$ . For the "double scaling limit" of the matrix model we always have  $L = -\partial_x^2 + u(x)$ ,  $A$  is some differential operator of the odd order  $2k + 1$ . In the first nontrivial case  $k = 1$  we have the Painlevé-1 (P-1) equation.

Only special real "separatrix" solutions of P-1 are important in the quantum field theory. By the conjecture of Novikov these "physical" solutions, which are analytically exceptional probably have much stronger symmetry then the other solutions but it is not proved until now.

Two asymptotic methods were developed in the previous paper - nonlinear semiclassics (or the Bogolubov-Whitham averaging method) and the linear semiclassics for the "Isomonodromic" method. The nonlinear semiclassics gives a good approximation for the general ("non-physical") solutions of P-1 but fails in the "physical" case.

In our paper the linear semiclassics for the "physical" solutions of the P-1 equations is studied. In particular connection between the semiclassics on Riemann surfaces and Hamiltonian foliations on these surfaces is established.



# INTEGRABLE BOUNDARY CONDITIONS FOR EVOLUTIONARY TYPE EQUATIONS

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March 6, 1995

## Abstract

Boundary value problems for integrable nonlinear partial differential equations are considered from the symmetry point of view. Families of boundary conditions compatible with the Harry-Dym, KdV and MKdV equations and the Volterra chain are discussed. We also discuss the uniqueness of some of these boundary conditions.

**Alfred Michel GRUNDLAND**

**CONDITIONAL SYMMETRIES FOR NONLINEAR PARTIAL  
DIFFERENTIAL EQUATIONS**

The purpose of this talk is to present the link between 1st order Backlund transformations and conditional symmetries admitted by 1st order systems of PDEs in  $p$  independent variables. The obtained results are applied to several examples of integrable and nonintegrable systems.

## Title

Inverse Problem, Linearization and Related Topics of Coupled Integrable Dispersionless Equation

## Authors

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## Abstract

Oono and one of the authors (K.K) found the following coupled integrable equation[1]:

$$\begin{aligned}iA_z - 2\alpha B_z A - 2\beta C_z B + \gamma(CB_z + C_z B) &= 0 \\iB_z - 2\alpha B_z B + 2\beta(2A_z A + CB_z) + 2\gamma B_z A_z &= 0 \\iC_z - 2\beta C_z C + 2\alpha(2A_z A + C_z B) + 2\gamma C_z A_z &= 0.\end{aligned}\tag{1}$$

Two special cases, such as  $A = -iq, B = C = ir$  for real  $q$  and  $r$  and  $A = -iq, B = C^* = -ir$  for real  $q$  and complex  $r$  with  $\alpha = \beta = 0$  and  $\gamma = 1$ , were solved by the inverse scattering method[1][2]. Very important property of these cases is that they are connected to the Sine-Gordon equation[3][4], and the Pohlmeyer-Lund-Regge Equations[5].

We will present an interesting linearization of Eq.(1), which is not based on the inverse problem. We will also discuss the inverse method and some related topics of Eq.(1).

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## **$p$ -adic description of chaos**

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$p$ -adic physical models [1,2] is the attempt to describe the reality with the aid of the number field  $Q_p$  which has many properties very different from real or complex case.

Quantum mechanics, where wave functions assume values in  $Q_p$ , was one of numerous  $p$ -adic models. The main problem of this theory is a probability interpretation of these wave functions. A new mathematical theory, a  $p$ -adic valued theory of probability was proposed in [2] to resolve this problem. As usual we consider a probability as a limit of relative frequencies  $\nu_n$  but with respect to another metric on the field of rational numbers  $Q$ . We play our game with the following evident fact. The only physical numbers are rational numbers. We can get in any experiment only finite fractions and not real, complex (or  $p$ -adic numbers). Then we can study these rational data with the aid of different mathematical methods.  $p$ -adics helps us to find some additional information about these rational numbers which we cannot find on the basis of real numbers. In particular, there exist such random sequences, where  $\nu_n$  oscillate between 0 and 1 with respect to the usual real metrics, but stabilize with respect to one of the  $p$ -adic metrics. This sequence are chaotic with respect to the usual theory of probability but they have a rigorous  $p$ -adic statistical structure.

This theoretical conception was realized in the series of the statistical computer experiments. A lot of statistical models (not only in physics) were constructed and we have seen the chaotic oscillations of relative frequencies with respect to the usual probability description and very quick statistical stabilization in the field of  $p$ -adic numbers.

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<sup>1</sup>This research was realized on the basis of Alexander von Humboldt-Stiftung

# ABSTRACT

## DOMAIN BOUNDARIES ASSOCIATED WITH SPONTANEOUS DEFORMATION OF ANTIFERROMAGNETIC CHAIN

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The 1D antiferromagnet (AFM) model is analyzed permitting the generalization of the Frenkel-Kontorova model to a system of two fields: the atomic displacement field and that of the atomic spin orientation. It is assumed that the equilibrium ordering of atomic masses caused by the mechanical interatomic interaction corresponds to the fully frustrated spin ordering. However, the magnetoelastic interaction gives rise to the spontaneous uniform deformation which results either in two-fold degenerated or non-degenerated AFM main state. In both cases, the magnetic domains can exist which are separated by domain boundary (DB) representing a soliton of the non-linear equation for the displacement field. It is shown that the DBs may be of two types. The DB of the first type retains the uniform spin distribution in the chain and is associated only with a kink of displacement field. There are two types of the kink (large and small). The above DB includes vacancy or crowdion, i.e. causes the non-elastic deformation of the  $2\pi$ -kink of the spin orientation field and gives rise to the deformation of the displacement field not violating the atomic ordering in the chain.

Abstract

It is shown that the equation which describes constant mean curvature surface via the generalized Weierstrass--Enneper inducing has Hamiltonian form. Its simplest finite--dimensional reduction is integrable and has two degrees of freedom. This finite-dimensional system admit  $S^1$ -action and classes of  $S^1$ -equivalence of its trajectories are in one-to-one correspondence with different helicoidal constant mean curvature surfaces. Thus the interpretation of well-known Delaunay and do Carmo--Dajzcer surfaces via integrable finite--dimensional Hamiltonian system is established.

## ABSTRACT

### SOLITON SOLUTIONS OF A NON-LOCAL SINE-GORDON MODEL

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Starting from the nonlinear equation describing a non - local interaction of dipoles along 1D chain [1] we derive a differential nonlinear equation which is a good approximation for the integral equation in the case of the short - range non - locality. The derived equation coincides with the generalized sine - Gordon equation proposed in Ref.[2]. The kink- like analytical solutions of the equation under consideration are known [2]. Using the asymptotic perturbation method we present analytical solutions of the two-parametrical dynamical soliton type and a bound state of kink-like solitons. It is found the solitons can transform into compactons in a special limiting case.

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## How the variational method give rise to false instabilities for 1-D solitary waves

D.J. Kaup and T.I. Lakoba

We address the problem of rigorous justification of the variational method (VM) applied to the study of small oscillations of 1D solitary waves (SW's). We determine the sufficient conditions in order for the VM not to give rise to *false instabilities* of such oscillations.

As is known, the VM requires an ansatz, i.e. a trial function for the exact SW which contains some parameters. The latter usually represent the SW's amplitude, width, phase, etc. and are allowed to vary with time. After inserting the ansatz into the corresponding Lagrangian density and integrating it over the spatial variable, one uses the resulting reduced Lagrangian to derive the equations of motion for the variational parameters. Thus, the study of SW dynamics, described by the original evolutionary equation (EE), is reduced to the study of a set of ODE's. This method had been used more or less successfully, until 1994 when B. Malomed and R. Tasgal (Phys.Rev.E, 49, 5787) discovered that when it was applied to the study of small oscillations of the soliton of the Massive Thirring Model (MTM), the method indicated that the soliton is unstable (in 1D). This contradicts the fact that the MTM is integrable by the IST, as well as numerical results.

We have found the reason why the VM failed in this case. Moreover, we will show that this phenomenon is entirely due to the linear structure of the MTM. We considered the MTM linearized on the background of the soliton and show that the eigenfunctions of the corresponding linear operator may be divided into two "spaces", distinguished by the sign of a certain inner product. If the variations in the ansatz contain components from both "spaces", then the VM could possibly give rise to a false instability. We can now put forward a method by which one is able to detect the possible occurrence of a false instability for a given 1D EE.

We also applied this method to the NLS and found that the VM will never give a false instability, provided that all the variations in the ansatz are, in a certain sense, orthogonal to the discrete spectrum of the NLS linearized about the soliton. Finally, we discuss the way of choosing an ansatz for an arbitrary 1D EE.



## ABSTRACT

### COMPACTONS AND PICONs IN MAGNETICALLY ORDERED MEDIA AND RELATION BETWEEN THEM AND INTERACTING BOSE- GAS PROBLEM

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Recently some attention has been focused on the new type of "exotic" soliton solution of nonlinear evolution equations - so called "compactons" and "picons". The field variable in compactons is nil exactly outside the domain of finite length and is proportional to trigonometric function (as in linear systems) inside of this domain. In picons the field is proportional to exponential function  $A \exp(-\alpha|x|)$  with definite amplitude. We investigate the new examples of compactons and picons in classical 1D uniaxial ferromagnetic with on-site anisotropy and strong anisotropic exchange interaction in the Ising limit and limit of XY- model. In Ising limit with easy-axis anisotropy there exists both compact domain walls and dynamic compactons with positive and negative frequency. In Ising ferromagnet with easy-plane anisotropy immobile picons exist which are similar to magnetic rotary waves or Lieb states in a nonideal Bose gas. In classical XY- model there are picon-type domain-walls in easy-axis ferromagnetic and compacton-type rotary waves in easy-plane case. We show that all these solutions represent the limit of analytical functions and study the envelope solitons, domain walls and rotary waves in the near-Ising and near-XY ferromagnet. The quantum treatment of new type solitons is advanced. It is well known that a small amplitude soliton is a classical analog of a bound state of bosons in Bose-gas with point two-particle interaction. A soliton in system with saturate nonlinearity is a classical analog of bound state of bosons in Bose-gas with point two-particle attraction and three-body repulsion. Exotic solitons are the classical analogs of bound state of bosons in Bose-gas with two-particle but complicated interaction. We propose the simple model of such interaction with two-body point attraction and repulsion on different distances. This quantum system is completely integrable in terms of Bethe ansatz. In Hartree approximation this quantum model transforms into the nonlinear equations with nonlinear terms involving spatial derivatives. Such equations arise in the theory of nonlinear waves in magnetically ordered and elastic media. The soliton solutions of these equations in some limits have an exotic form.

## SECOND HARMONIC OF NONLINEAR RESPONSE OF MAGNETS

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We present the results of theoretical and experimental studies of second harmonic of magnetization for magnets submitted in parallel static and alternating  $h(t)$  magnetic fields. The corresponding nonlinear response has been investigated experimentally in dilute solid paramagnets and their solutions [1] as well as in concentrated exchange magnets [2,3].

The nonlinear response is analyzed on the base of two supplementary approaches [4]. In the first one nonlinear susceptibility  $\chi_2(\omega)$ , determined by the second order term in expansion of magnetization in  $h(t)$ , is investigated. It is expressed in terms of third-order spin Green's function. Using this relation and symmetry considerations, one can determine the general character of  $\chi_2(\omega)$  dependence, namely, to determine asymptotic behavior and make definite conclusions on number of zeroes of  $\text{Re } \chi_2(\omega)$ ,  $\text{Im } \chi_2(\omega)$ .

In the second approach kinetic equation for spin matrix density is used. It is analyzed in framework of perturbation theory on spin-spin or spin-lattice interactions. As a result the explicit expressions for  $\chi_2(\omega)$  can be obtained.

The simple physical systems which show the main types of the nonlinear response are considered. They are a paramagnetic complex with  $S=1/2$  in liquid whose relaxation is due to spin-rotation coupling and complex with hyperfine interaction.

The available experimental data on the nonlinear response of dilute paramagnets are analyzed.

The recent results of studies of this response for a cubic ferromagnet in the critical region above  $T_c$  [2] and antiferromagnet  $\text{La}_2\text{CuO}_{4+x}$  [3] are presented.

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# Solutions of Some Soliton Equations via Wavelet Basis

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TAIWAN

## Abstract

Wavelet theory develop very rapidly in this decade. Recently G. Beylkin, R. R. Coifman, V. Rokhlin developed an algorithm of wavelet basis for the computation of Calderon-Zygmund operators. Hilbert transform is the most typical example. On the other hand, it is known that the Hilbert transform is closely related to some soliton equations, implicitly or explicitly, e.g. nonlinear Schrodinger equation(NLS), derivative nonlinear Schrodinger equation(DNLS), Benjamin-Ono equation, etc. Here we apply the Beylkin-Coifman-Rokhlin algorithm to the numerical computation of some non-soliton solutions of NLS and DNLS. We will present preliminary results.

# THE UPPER OCEAN: SYMMETRY BREAKING, COHERENT STRUCTURES, AND MIXING

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## Abstract

Persistent wind-driven patterns on the surfaces of natural water bodies of all sizes, from the oceans to small ponds, are manifestations of an underlying convective mechanism causing large scale mixing. The patterns, known as windrows or "Langmuir circulation", typically are series of elongated streaks nearly parallel to the local wind direction, and with a hierarchy of spacings ranging from meters to hundreds of meters. The phenomenon is mechanically driven, and heat transfers are associated consequences rather than causes. Experimental evidence suggests that the largest scales are comparable to the depth of the water layer, or, in the case of density stratified bodies of water such as large lakes and the oceans, to the depth of the seasonal thermocline. They represent the most coherent structures in the ocean mixed layer on time scales of hours, and are believed to be the most powerful mixing mechanism in the upper layers of the ocean and lakes. We describe the theory of the phenomenon, symmetries and bifurcation sequences, massive computation utilizing large-eddy-simulation to treat the presence of the coherent structures in fully developed turbulent mixed layers, and order parameters for the study pattern formation in extended regions of the ocean. The possibility of remote sensing, by satellite, of sea surface patterns due to Langmuir circulation will be discussed, together with the utility of such observations to the physics of climate and climate change.

# LONG-PULSE EXPERIMENTS IN GAS: SPECTRAL TRANSFORM AND INTERPRETATION

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The nonlinear theory of stimulated Raman scattering of high energy long laser pulses in two-level media is constructed and provides a unified global interpretation of the experiments of Drühl, Wenzel and Carlsten (Phys. Rev. Lett., **51**, 1171 (1983)). It is proved that the model, resulting from the slowly varying envelope approximation, and including phase mismatch and detuning from Raman resonance, is integrable by the spectral transform method for arbitrary boundary values. The essential variable in the theory is the phase of the Stokes pulse relatively to the pump. For long duration input pump pulses and when pump and Stokes pulses experience a rapid  $\pi$ -phase shift, an anomalous spike of pump radiation is created in the pump depletion zone. It is shown in particular that the decay of Raman spikes is related to the *velocity* of this  $\pi$ -phase shift. For fast flips, the possibility of observing spike overflow is established and with multiple phase flips, a multiple-spike phenomenon is produced, suggesting new experiments. Last, the number of optical phonons participating to the interaction is explicitly calculated.

# BINARY NONLINEARIZATION OF LAX PAIRS

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## Abstract

Support that the isospectral ( $\lambda_{t_n} = 0$ ) zero curvature equations  $U_{t_n} - V_x^{(n)} + [U, V^{(n)}] = 0$ ,  $n \geq 0$ , of Lax pairs

$$\phi_x = U\phi = U(u, \lambda)\phi, \quad \phi_{t_n} = V^{(n)}\phi = V^{(n)}(u, \lambda)\phi, \quad n \geq 0 \quad (1)$$

determine a soliton hierarchy

$$u_{t_n} = K_n = JG_n = J \frac{\delta H_n}{\delta u}, \quad n \geq 0. \quad (2)$$

From the spectral problem and adjoint spectral problem  $\phi_x = U\phi$ ,  $\psi_x = -U^T\psi$ , we can calculate that

$$\frac{\delta \lambda}{\delta u} = \frac{\langle \phi \psi^T, \frac{\partial U}{\partial u} \rangle}{-\int_{-\infty}^{\infty} \langle \phi \psi^T, \frac{\partial U}{\partial \lambda} \rangle dx}, \quad \text{where } \langle A, B \rangle = \text{tr}(AB).$$

On introducing distinct eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_N$ , make the special symmetry constraints

$$K_m = JG_m = J \sum_{j=1}^N E_j \frac{\delta \lambda_j}{\delta u} \quad \text{or} \quad G_m = \sum_{j=1}^N E_j \frac{\delta \lambda_j}{\delta u}, \quad E_j = - \int_{-\infty}^{\infty} \langle \phi_j \psi_j^T, \frac{\partial U}{\partial \lambda_j} \rangle dx \quad (m \geq 0). \quad (3)$$

The Bargmann constraint requires the  $G$ -vector field to be a potential function not including any potential differential (usually  $G_0$ ) and thus we may find an explicit nonlinear expression for the potential

$$u = f(\phi_1, \phi_2, \dots, \phi_N; \psi_1, \psi_2, \dots, \psi_N). \quad (4)$$

After instituting (4) into Lax pairs and adjoint Lax pairs, we obtain two nonlinearized (or constrained) systems

$$\text{spatial part: } \phi_{jx} = U(f, \lambda_j)\phi_j, \quad \psi_{jx} = -U^T(f, \lambda_j)\psi_j, \quad j = 1, 2, \dots, N; \quad (5)$$

$$\text{temporal part: } \phi_{jt_n} = V^{(n)}(f, \lambda_j)\phi_j, \quad \psi_{jt_n} = -(V^{(n)})^T(f, \lambda_j)\psi_j, \quad j = 1, 2, \dots, N \quad (n \geq 0). \quad (6)$$

In order to obtain integrals of motion for (5) and (6), we choose a solution  $V$  to the adjoint representation equation:  $V = \sum_{i \geq 0} V_i \lambda^{-i}$ ,  $V_x = [U, V]$ . We also have  $V_{t_n} = [V^{(n)}, V]$  when  $u_{t_n} = K_n$ . Therefore  $F = \frac{1}{2} \text{tr} V^2$  is a common generating function for integrals of motion of (5) and (6). Noticing  $F = \sum_{n \geq 0} F_n \lambda^{-n}$ , we get a series of integrals of motion:  $\{F_n\}_{n=0}^{\infty}$ , which may be proved to be involutive. Moreover  $u = f(\phi_j; \psi_j)$  gives an involutive solution to the  $n$ -th soliton equation  $u_{t_n} = K_n$  once  $\phi_j, \psi_j$  solve (5) and (6), respectively. This kind of involutive solutions also exhibits a kind of separation of independent variables  $x, t_n$  for soliton equations. Some examples are carried out.

# Lie symmetries for differential difference equations.

D. Levi

## Abstract:

We review the results obtained up to now for integrating differential difference equations using the methods of Lie group theory. At first we consider just the continuous symmetries. We will apply the obtained algorithms to the case of the Toda-Lattice equation and to the classification of discrete dynamical systems. At the end we will show some preliminary results on how such results could be extended to the case of discrete symmetries.

# Long-Waves in Nonlinear Dispersive Systems: Multiple-Time Solutions and the KdV Hierarchy

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## Abstract

We study long-waves in nonlinear dispersive systems from the point of view of a multiple-time reductive perturbation method. Then, in the case of the Boussinesq equation, as a consequence of the elimination of the secular producing terms through the use of the Korteweg-de Vries hierarchy, we show that its solitary-wave is a solitary-wave satisfying simultaneously all equations of the Korteweg-de Vries hierarchy, each one in an appropriate time variable.



# PROPAGATION OF A SOLITON IN A PERIODICALLY MODULATED NONLINEAR WAVEGUIDE

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We consider the nonlinear Schrödinger (NLS) equation whose dispersion coefficient is a periodic function of the propagation distance  $z$ :

$$iu_z + \frac{1}{2}(1 + \epsilon \sin z) u_{\tau\tau} + |u|^2 u = 0, \quad (1)$$

where the coefficient  $\epsilon$  characterizes the variable part of the dispersion coefficient, and  $\tau \equiv (t - z)/V_{gr}$ ,  $V_{gr}$  being the mean group velocity of the carrier wave. Equation (1) is a general model of a periodically inhomogeneous nonlinear waveguide, which has at least two important physical applications: a nonlinear optical fiber with a periodically modulated diameter (which can be easily fabricated by means of the available technology), and natural guides for internal waves in the ocean, which are usually strongly inhomogeneous. In the recent work [1], evolution of a soliton governed by (1) was considered semianalytically by means of the variational approximation. In this approximation, the soliton is given a single internal (vibrational) degree of freedom, for which an effective equation of motion was derived in [1]. Numerical simulations of the resulting ordinary differential equation which describes this motion predict a sudden decay of the soliton into radiation when  $\epsilon$  exceeds a critical value, which is close to  $\frac{1}{4}$  and weakly depends upon the initial soliton's energy  $E$ . The effect predicted may be regarded as a manifestation of a nonlinear resonance between internal vibrations of the soliton and the periodic modulation (in optical fibers, the resonance may be realistic for subpicosecond solitons).

The aim of this work is to check the possibility of abrupt transformations of the soliton in the model (1) by means of systematic direct numerical simulations. We find that, when the soliton is sufficiently broad, it remains quite stable, emitting radiation at a very small rate. However, when the soliton's dispersion length becomes comparable to the modulation period (so that a resonance may be expected), the soliton suddenly splits into a pair of *secondary* solitons, which is accompanied by a burst of radiation. This happens when  $\epsilon$  exceeds a critical value, which, in order of magnitude, coincides with that predicted in [1], despite the fact that the simplest variational approximation considered in [1] did not take into account the possibility of the splitting of the soliton into two. The secondary soliton, being essentially broader, proves to be more stable than the initial one. With further increase of  $\epsilon$ , we have found several *stability islands*, in which the soliton partially retrieves its stability, although it demonstrates persistent internal vibrations and more conspicuous emission of radiation. The existence of a complicated pattern of the stability islands resembles the well-known effect of alternating regions of elastic and inelastic soliton-soliton collisions in nonintegrable systems [2]. The splitting of a soliton into the pair of secondary ones may find a practical application in photonics.

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DFUL-1/06/95

# TOPOLOGICAL FIELD THEORY AND NONLINEAR $\sigma$ -MODELS ON SYMMETRIC SPACES

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## Abstract

*We show that the classical non-abelian pure Chern-Simons action is related to nonrelativistic models in (2+1)-dimensions, via reductions of the gauge connection in Hermitian symmetric spaces. In such models the matter fields are coupled to gauge Chern-Simons fields, which are associated with the isotropy subgroup of the considered symmetric space. Moreover, they can be related to certain (integrable and non-integrable) evolution systems, like the Ishimori and the Heisenberg model. The main relevant classical and quantum properties of these systems are discussed in connection with the topological field theory and the condensed matter physics.*

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ABSTRACT :

EXPERIMENTAL INVESTIGATION OF PHASE STATES IN  
PARALLEL-SERIAL JOSEPHSON ARRAYS.

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Recently developed theory of parallel-serial quantum interferometers [1,2] predicts the existence of unusual phase states resulted in the multiple branches of critical current as a function of external magnetic flux. These states arise as a consequence of double independent means at which the phase of weakest Josephson junction is mapped into the phase of other serial-connected junctions. Each state of the whole system has its own quantum number and separates from another by energy barrier the value of which depends on the parameters of Josephson junctions.

The concept of phase states is not generally accepted yet. Nevertheless, it gives the possibility to describe the large number of physical phenomena in quantum Josephson arrays, including ones composed of weak-connected granules of high temperature superconductors.

In this paper we present the experimental evidence of existence of the phase states. The measurements were performed on four-contact interferometers prepared in planar trilayer Nb/Al - AlO<sub>x</sub>/Nb technology. The experimental data are compared with the numerical results obtained on the base of Lewandowski phase-state concept.

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1. S.J.Lewandowski, Phys.Rev.B 45, 7776 (1992).

# ON THE EXISTENCE OF SELF-SIMILAR STRUCTURES IN THE RESONANCE REGION

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A nonlinear Hamiltonian system with two degrees of freedom is considered which depends on the parameter  $\epsilon$  and is integrable at  $\epsilon = 0$ . It is shown that in the case of general position, for any resonance region there exists a set  $E$ , having continual cardinality, of critical values of the parameter  $\epsilon$  such that at any critical value of  $\epsilon$  the system under consideration has a self-similar structure of the type "islands around islands". In this situation, the behaviour of solutions in each cell of this structure is determined in the first approximation by a certain standard Hamiltonian. But the fine structure of each cell is described by additional terms of the Hamiltonian which are considered to be small perturbation.

# SUPERCONDUCTING AND SPIN GLASS INTERPLAY IN COUPLED LAYERED STRUCTURES

L.V. MERCALDO

## ABSTRACT

For studying the mutual effects of superconductivity and magnetism in coupled layered structures, multilayers of Nb (superconducting) and CuMn (spin glass) with different Mn concentrations were grown on Si (100) substrates by dc triode magnetron sputtering in the same deposition run. In all samples Nb layer thickness was kept constant (24.5 nm), while varying CuMn layer thickness between .3 nm and 18.4 nm. The superconducting transition temperature showed a pronounced nonmonotonic dependence on the spin glass CuMn layer thickness. This behaviour was compared with Hauser et al. and Radovic et al. theoretical models. A good description of our experimental data is obtained using the second one, which foresees changing phase difference values between neighboring superconducting layers.

# On Integrable Nonhomogeneous Systems of Hydrodynamic Type with Quadratic Nonlinearity

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We study Hamiltonian and integrability properties of some special class of non-linear nonhomogeneous systems of hydrodynamic type with quadratic nonlinearity ([1]–[3]):

$$u_t^i = a^i u_x^i + \sum_{j,k} b_{jk}^i u^j u^k + \sum_k c_k^i u^k, \quad (1)$$

where a summation over repeating indices is not assumed,  $a^i, b_{jk}^i$  and  $c_k^i$  are constant tensors,  $i, j, k = 1, \dots, N$ .

There are a number of well-known integrable systems among (1) such as the  $N$ -wave equations (for example, integrable real-valued exact resonance system of parametric interaction of three wave packets in nonlinear optics ( $N = 3$ ) is a special case of (1)), the Korteweg-de Vries equation considered as evolution system with respect to the space variable  $x$  (see [1–3]) and others.

Some new integrable systems of the type (1) are found. For example, the following theorem is proved.

**Theorem.** The nonhomogeneous system of hydrodynamic type

$$u_t^i = a^i u_x^i + u^i \sum_k (a^i - a^k) u^k, \quad (2)$$

where  $a^i \neq a^j$ , if  $i \neq j$ ,  $i, j = 1, \dots, N$ , is integrable and equivalent to the homogeneous diagonal Hamiltonian system of hydrodynamic type

$$v_t^i = \left( \sum_k (a^i - a^k) e^{v^k} \right) v_x^i \quad (3)$$

after a combination of a reciprocal transformation and some changes of the fields and independent variables.

This work was partially supported by the Russian Foundation of Fundamental Researches (Grant No. 94-01-01478) and the International Science Foundation (Grant No. RKR000).

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LUIGI MINELLI

Abstract.

A large amount of literature is presently aimed to the set up of clinical indicators deduced from time-EEG series by advanced spectral techniques or by methods commonly employed in non-linear dynamics analysis. To draw relevant information on a system with more of a billion of degrees of freedom from an extremely limited sampling, also contaminated by noise, is, in principle, an hazardous challenge. On the other hand, the relatively low measured dimensionality of brain activity encourages similar approaches. So we have tested a denoising algorithm and tried to explain this phenomenon in terms of synchronization, using a map inspired to the integrate and fire neuron model. Since the use of more or less realistic neural networks appears also a promising tool for the parametrization and classification of the rhythms we have developed the map in the framework of a formalism apt to include the main models.

# NON-FUCHSIAN EXTENSION TO THE PAINLEVÉ TEST

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**Abstract** – We consider [6] closed form meromorphic solutions of nonlinear ordinary differential equations, which make their linearized equation non-Fuchsian at a movable singularity, Fuchsian at infinity, without any other singular point. When the nonlinear ODE possesses movable logarithms, a perturbation à la Poincaré detects them sooner than the perturbative (Fuchsian) Painlevé test [4, 2]; indeed, we can investigate the point at infinity, because the particular solution which we consider is *global*, while the series of the perturbative Painlevé test is *local* by definition.

Two examples are presented: an equation of Bureau [1]  $u'''' + 3uu'' - 4u'^2 = 0$ , the sixth order dynamical system governing the Bianchi IX cosmological model in the logarithmic time [3, 5].

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# NON-FUCHSIAN EXTENSION TO THE PAINLEVÉ TEST

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**Abstract** – We consider [6] closed form meromorphic solutions of nonlinear ordinary differential equations, which make their linearized equation non-Fuchsian at a movable singularity, Fuchsian at infinity, without any other singular point. When the nonlinear ODE possesses movable logarithms, a perturbation à la Poincaré detects them sooner than the perturbative (Fuchsian) Painlevé test [4, 2]; indeed, we can investigate the point at infinity, because the particular solution which we consider is *global*, while the series of the perturbative Painlevé test is *local* by definition.

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- [6] M. Musette and R. Conte, préirage SPEC 94/118.

# Darboux Transformations for Integrable Lattice Systems

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## Abstract

A framework for a general description of Darboux and binary Darboux transformations for Lax representations of discrete integrable systems is presented. The Lax equations are regarded as systems in the algebra of shift operators which is embedded in an algebra of pseudo-difference symbols. Gauge transformations are given by operators satisfying a dressing equation in this space. Solutions are found which are parametrized by (adjoint) eigenfunctions of the Lax system. A natural application of this framework involves the lattice (m)KP hierarchy as considered by Kupershmidt [Discrete Lax equations and differential-difference equations, *Astérisque* 123 (1985)]. Two  $r$ -matrices corresponding to simple decompositions of the algebra of pseudo-difference symbols give rise to the discretized KP and mKP equations which represent the auto-Bäcklund transformations of the continuous KP/mKP hierarchies. Reductions to finitely many fields are discussed. The simplest representatives are the Toda hierarchy (KP) and the relativistic Toda hierarchy (mKP). The Darboux transformations leave the special parametrization of the corresponding Lax operators invariant so that solutions can be obtained.

# NONLINEAR WAVES GENERATED BY INSTABILITIES

## IN PRESENCE OF A CONSERVATION LAW

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In the last decades, the spontaneous formation of spatially inhomogeneous patterns was an object of extensive investigations. A large amount of phenomena has been described by means of the generalized Ginzburg-Landau equation. However, this equation is definitely not valid in the case of a long-wavelength instability in presence of a *conservation law* that precludes the growth of spatially homogeneous disturbances. For instance, in the case of wavy instabilities in a layer of an incompressible fluid with a free surface, a homogeneous change of the depth of the layer is impossible because of the conservation of the fluid volume.

In the present lecture we consider the order parameter equations replacing the Ginzburg-Landau equation in systems with the conservation law. It turns out that the most generic equation for the amplitude function  $U(X, T)$  is the *dissipation-modified Korteweg-de Vries equation*

$$U_T + 6UU_X + U_{XXX} + \epsilon(U_{XX} + U_{XXX} + D(U^2)_{XX}) = 0, \quad (1)$$

which has been derived in many physical problems like modulational instability of periodic waves, instability of a liquid film flowing down a slightly inclined plane, convection problems and so on. In special cases, according to symmetry properties of the underlying physical properties, this generic equation is replaced by the Kawahara equation, Kuramoto-Sivashinsky equation, Cahn-Hilliard equation etc.

We concentrate on the stability problem for spatially periodic nonlinear waves determining their "Busse balloon". In some cases, the nonlinear equations governing the modulational instability are obtained. The problem of "blow-up" is discussed.

# N-soliton solution of Harry Dym equation by Inverse Scattering method

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## Abstract

N-soliton solution of Harry Dym equation was found by means of the inverse scattering method. But it was solved only one-soliton solution in exact form. In this lecture we study N-soliton solution of Harry Dym equation in exact form by inverse scattering method.

# On Periodic Solutions of Nonlinear Wave Equations, the Bilinear Transform and Nonlinear Superposition

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Partial differential equations that describe nonlinear wave propagation and which can be solved exactly by techniques based on the inverse scattering transform, have become familiar landmarks in the last twenty five years. One aspect of this work has been the recognition of nonlinear superposition principles which - in one guise or another - characterise many of the solutions to these integrable evolution equations. Typically, we mention the use of Bäcklund Transformations to generate a new solution in terms of two known solutions by algebraic means. Some recent studies have focused on those *nonlinear* stationary periodic waves which have the remarkable property that they can be formed by superposing solitary waves in a *linear* fashion. The first result of this kind was obtained by Morikazu Toda for the cnoidal wave of the KdV equation. Since then, many other nonlinear evolution equations have been shown to possess this same property which reveals the beautifully simple structure of certain periodic solutions as an array of repeated solitary waves. This elegant structure can be interpreted as a very special *nonlinear superposition principle*. Interestingly, this property is not confined to the class of completely integrable equations, but is shared, for example, by the partially integrable RLW equation and the nonintegrable Burgers equation. We develop these ideas using Hirota's bilinear transformation method and Riemann theta functions, and results are presented for completely integrable equations (KdV, ILW, BO), the RLW equation and the Burgers equation. We also outline the results of some recent investigations which generalise the bilinear - theta function procedure, thereby extending the work of Akira Nakamura in this area. The general theory provides a powerful, yet practical analytic tool with which to investigate periodic solutions for a broad class of nonlinear wave equations. By taking the mKdV equation as our vehicle, we are able to give new expressions for Jacobi's elliptic functions  $cn$ ,  $sn$  and  $dn$  in terms of theta functions. These differ markedly from the classical definitions of elliptic functions as ratios of theta functions and turn out to be more useful in the study of nonlinear periodic waves.

# INTEGRABLE CHERN-SIMONS GAUGE FIELD THEORY IN 2+1 DIMENSIONS

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## ABSTRACT

The classical spin model in planar condensed media is represented as the  $U(1)$  Chern-Simons gauge field theory. When the vorticity of the continuous flow of the media coincides with the statistical magnetic field, which is necessary for the model's integrability, the theory admits zero curvature connection. This allows me to formulate the model in terms of gauge - invariant fields whose evolution is described by the Davey-Stewartson (DS) equations. The Self-dual Chern-Simons solitons described by the Liouville equation are subjected to corresponding integrable dynamics. As a by-product the 2+1-dimensional zero-curvature representation for the DS equation is obtained as well as the new reduction conditions related to the DS-I case. Some possible applications for the statistical transmutation in the anyon superfluid and TQFT are briefly discussed.

MIRAMARE - TRIESTE

April 1995

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# Secular-Free Perturbation Theory and the Nonlinear Schrödinger Hierarchy

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## Abstract

We use the multiple scale method with an infinite sequence of slow time variables to study the modulation of dispersive waves governed by a nonlinear partial differential equation. In this context, as is well known, the Nonlinear Schrödinger equation (NLS) appears as the condition for eliminating the secularities at the lowest relevant order. Then, by considering a traveling-wave solution to the NLS equation, we show that, as a consequence of the requirement of absence of the corresponding secular terms in each order of the perturbative scheme, the equations of the NLS hierarchy do play an important role in the description of such waves.

F. Pempinelli, M. Boiti, J. Leon

ABSTRACT

The initial value problem for a nonlinear discrete system with singular dispersion relation and arbitrary boundary conditions is solved by extending the theory of the spectral transform related to the Ablowitz-Ladik spectral problem.

Backlund gauge transformations are also extended to this case.



# Lax pairs and exact solutions from Painlevé analysis

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## Abstract

A new Riccati variable allows us to simplify the process of recovering the Lax pairs of nonlinear partial differential equations such as modified Korteweg-de Vries and Broer-Kaup from Painlevé analysis. This new variable can be expressed in terms of the original Weiss-Tabor-Carnevale (WTC) singular manifold. This then leads us to revise our understanding of the singular manifold method.

The generalisation of the singular manifold method necessary to handle such equations is one which corresponds to the summation of infinite WTC expansions for certain choices of arbitrary data. This is of course a natural extension of the usual truncation process, and involves only one singular manifold. We are then able to give a new and more consistent definition of "singular manifold equation." A corollary of our analysis is a direct proof of the convergence of infinite WTC expansions under certain constraints.

In addition, the approach developed here allows us to obtain from Painlevé analysis a larger class of exact solutions than was possible hitherto. Again, our analysis greatly simplifies the recovery of such solutions.

## Dynamics of blood pressure waves in large arteries

M. REMOISSENET

We present a nonlinear quasi-dimensional model which describes blood pressure propagation in large arteries. In the limit of an ideal fluid and for slowly varying arterial parameters a Boussinesq-type equation is obtained. Numerical simulations reveal the influence of changing the diameter and the Young modulus, on the pressure pulse shape. Physiological features such as "peaking" and "steepening" show that the pulse can be seen as a wave whose shape evolves between a solitary and a shock wave.

# ON CONSERVATION LAWS AND SYMMETRIES FOR NONLINEAR DIFFERENTIAL EQUATIONS

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We discuss the relationship between generalized (Lie-Backlund) symmetries and local conservation laws for partial differential equations. We consider the approach based on the Noether operator relation which allows the association of conserved quantities with symmetries for a large class of differential equations regardless of the existence of a well-defined Lagrangian function. Among the equations of the class are many physically interesting equations: Korteweg-de Vries, Kadomtsev-Petviashvili, Boussinesq equations, nonlinear heat equation, nonlinear diffusion equation, regularized long-wave equation, Euler equations, Navier-Stokes equations, etc.

## Generalized Ermakov Systems

COLIN ROGERS

**Abstract.** New extensions of the classical Ermakov system both with regard to dimension and order are presented. These possess an underlying linear structure inherited from that recently established for the one-dimensional Ermakov system. Auto-Bäcklund transformations and generation of solutions via nonlinear superposition principles are discussed. Particular classes of solution of the Ernst and Bianchi systems are generated via constrained Ermakov systems. Remarkably, the recursion operator structure associated with a hierarchy of higher-order Ermakov systems coincides, up to a sign, with that associated with the AKNS linear representation of soliton theory.

Patchwork approach to nonlinearity, inverse problems, and  
interdisciplinarity?

From the middle of nineteenth century, nonlinear aspects of  
physical phenomena appeared as special features on patchworks whose  
main texture is linear. Are times coming when they may be related  
together? In nonlinear inverse problems, answer to this question,  
together with overdetermination, may lead to a solving method.

Friendly

Pierre C. Sabatier

PAOLO MARIA SANTINI

- 1) Multiscale expansions in Physics and the Nonlinear Schroedinger hierarchy (with A.Degasperis and S.Manakov).
- 2) Solitons, compactons, and an inverse acoustic problem (with A.Fokas).

Abstract.

- 1) We consider natural phenomena modelled by systems of nonlinear PDE's, dispersive in the linear limit. We show that the slowly varying effects of the nonlinearity on the amplitude of a dispersive monochromatic wave are described by the nonlinear Schroedinger hierarchy of PDE's. We apply these results to Fluid Dynamics and Plasma Physics.
- 2) Long waves of moderate amplitudes are described by a nonlinear PDE introduced by Fokas -Fuchssteiner, and recently rediscovered by Camassa-Holm. This equation possesses a rich zoo of solutions, including: solitons, compactons and peakons. We present the solution of the initial-value problem for this equation, associated with a new inverse acoustic problem.

**The Geometry of the LKR System:  
Application of a Laplace Transformation to Ernst and Bianchi-type Equations**

**WOLFGANG K. SCHIEF**

**Abstract.** Classical geometric systems of Darboux, Bianchi and Weingarten are identified as members of the integrable Loewner-Konopelchenko-Rogers system. In this connection, constraints which preserve integrability are presented along with compatible solution generation techniques. In particular, a novel Laplace-type transformation is discussed, leading to sequences of *nonlinear* Ernst and Bianchi-type equations akin to those obtained in classical Stokes-Beltrami Theory for *linear* equations.

# DEFECT-LIKE SOLUTIONS OF 2D SINE-GORDON EQUATION AS A MODEL OF INHOMOGENEOUS STATES IN LARGE AREA JOSEPHSON JUNCTIONS

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The singular defect-like solutions of 2D elliptic SG equation

$$\Delta\varphi = \sin(\varphi), \quad (1)$$

which were found recently by computer methods [1,2], give a unified approach to physics of defects in condensed matter. Now a full classification problem has been solved [2]: the defect-like solutions of eq.(1) may be of two basic types - "source" and "vortices". In this report the theory of defects will be applied to large area Josephson junctions with sizes much greater than the Josephson penetration depth. For the large junctions the typical phenomenon is strong spatial localizations of currents and fields which can be approximated by distributions of the defects.

For large area junctions the source defect-like solution associates with the point-like injection of bias currents and may be interpreted as a system of snugly packing ring fluxons with common center in injection point. The simplest topological vortex solution corresponds to entrance of a unit Abrikosov vortex from superconductive strip into the junction and consists of a semi-infinite fluxon line starting from the entrance point. More complex distributions are composed from these two basic types of defects. In term of eq.(1) we formulate the singular boundary value problem which describes a stationary superconductive mixed state of the junction with defects.

Under influence of external perturbations the mixed states became unstable which result in dynamic resistive state of the junction. The instability will be investigated within the scope of damped and driven generalization of eq.(1)

$$-\varphi_{tt} - \gamma\varphi_t + \Delta\varphi = \sin(\varphi) + f(t). \quad (2)$$

In particular, driven instability of isolated defects gives rise to the new dynamical structures such as spirals and target patterns. For rf perturbation  $f(t)$  the defect configurations have high quality resonant properties which cause efficient destruction of the defects with emission of waves and generation of a turbulent resistive state of the junction.

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# MULTYPOINT CORRELATION FUNCTIONS IN ONE-DIMENSIONAL IMPENETRABLE BOSE-GAS

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## ABSTRACT

We consider the quantum one-dimensional impenetrable Bose-gas with repulsive interaction. This model is described by quantum Nonlinear Schrödinger equation. In the number of recent works the two-point correlation functions in this model were explicitly calculated. It was shown, that correlation functions can be expressed in terms of the solution of classical Nonlinear Schrödinger equation.

In given work we develop this approach to the calculation of multipoint correlation functions. We obtain the representation for correlators in terms of Fredholm determinant and prove, that this determinant can be expressed in terms of the solution of the system of vector Nonlinear Schrödinger equations. The correlation function itself has the properties of the  $\tau$ -function of this system.

# Dynamics of Molecular Crystals: Simulations vs. Experiment.

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Molecular dynamics simulation and harmonic analysis are combined with neutron scattering experiments to determine atomic dynamics in molecular crystals. The cases examined comprise lattice vibrations in a zwitterionic amino-acid (1), jumps between equivalent sites in a peptide (2), alkane chain diffusion in urea clathrate inclusion compounds (3), local vibrational anharmonicities in hydrogen-bonded systems (4), and slow, whole-molecule oscillations in pure and doped polyacetylene (5,6).

## References

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# ON THE DARBOUX INTEGRABLE NONLINEAR HYPERBOLIC EQUATIONS

V.V. Sokolov

It is well known that there exist two different kinds of integrable one-component nonlinear hyperbolic equations of the form

$$u_{xy} = F(x, y, u, u_x, u_y). \quad (1)$$

These are equations of the Liouville and the sin-Gordon types. The first type equations are called Darboux integrable. The term "Darboux integrability" means the existence of functions  $P(x, y, u, u_x, u_y, \dots)$  and  $Q(x, y, u, u_x, u_y, \dots)$  such that  $P$  is a function of  $x$  and  $Q$  is a function of  $y$  for any solution  $u(x, y)$  of (1). It is interesting to note that any Darboux integrable equation generates a pair of Miura type differential substitutions linking nonlinear evolution equations.

Recently a relationship between the Darboux integrability and the Laplace invariants of the linearization operator

$$L = D_x \circ D_y - \frac{\partial F}{\partial p} D_x - \frac{\partial F}{\partial q} D_y - \frac{\partial F}{\partial u}, \quad p = u_x, \quad q = u_y \quad (2)$$

of (1) have been established. The main Laplace invariants of the operator (2) are given by

$$H_0 = D_x \left( \frac{\partial F}{\partial p} \right) - \frac{\partial F}{\partial p} \frac{\partial F}{\partial q} - \frac{\partial F}{\partial u}, \quad H_1 = D_y \left( \frac{\partial F}{\partial q} \right) - \frac{\partial F}{\partial p} \frac{\partial F}{\partial q} - \frac{\partial F}{\partial u}.$$

The other Laplace invariants can be found recurrently from the following formula

$$D_x D_y (\text{Log}(H_i)) = H_{i+1} + H_{i-1} - 2H_i, \quad i \in \mathbb{Z}. \quad (3)$$

It turns out that (1) is Darboux integrable iff there exist  $n \geq 1$  and  $m \leq 0$  such that  $H_n = H_m = 0$ . This fact gives us a powerful criteria of the Darboux integrability and allows to describe the Lie algebra of generalized symmetries for any Darboux integrable equation.

# On the existence and stability of solitary wave solutions in discrete nonlinear Schrödinger systems

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The physical descriptions of many nonlinear systems result in discrete models. Typical examples are: optical pulse propagation in arrays of coupled optical waveguides, proton dynamics in hydrogen-bonded chains, transport of excitation energy in biophysical systems, Scheibe aggregates, Hubbard model, electrical lattices, DNA dynamics, molecular crystals, and so on. In this contribution we investigate some new techniques for the existence and stability of discrete solitary waves, based on a one-dimensional discrete nonlinear Schrödinger equation. We start with a discussion of stationary states by examining possible variational principles and generating functions. The latter technique makes use of solutions of a continuous difference equation and allows for solutions with different symmetry properties. Actually, whole families of stationary solutions are found. In the second part we investigate the stability properties of the newly found solutions and derive complementary variational principles and a so-called N-theorem for stability which is easy to handle. The analytical calculations are supplemented by numerical simulations which also exhibit the nonlinear evolution of instabilities. Finally, we briefly comment on extensions to two and three space dimensions.

## THE QUANTUM EASY-PLANE FERRO- AND AN-

### TIFERROMAGNET

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The Berezinskii-Kosterlitz-Thouless (BKT) phase transition is commonly supposed to show up in two-dimensional magnetic systems with easy-plane anisotropy. Their study has been usually approached by the classical XY model, neglecting the role of the out-of-plane spin component. However, the out-of-plane components must be included for describing real compounds, and their fluctuations cannot be neglected when quantum effects are to be included, quantum spins being intrinsically three-component objects. Therefore, we have approached the easy-plane XXZ model by the effective Hamiltonian method, that reduces the quantum thermodynamics to the investigation of an effective classical model with temperature dependent renormalized interaction parameters. This makes it possible to obtain explicit evaluations of the quantum thermodynamic quantities by Monte Carlo simulations or any other known classical method. As expected, quantum fluctuations reduce the effective spin length, and thus the exchange interaction, resulting in a lowered BKT transition temperature  $T_{\text{BKT}}$ . A more striking effect that we can forecast from our approach is that quantum fluctuations are in competition with the easy-plane anisotropy, and could give rise to an *isotropization* of the effective classical model, suppressing the BKT transition. We have studied the easy-plane XXZ model on a square two-dimensional lattice. For the classical model the thermodynamic properties of the system have been investigated in a large range of temperatures by means of Monte Carlo simulations for different sizes of the lattice. This allows to locate

# MULTICOMPONENT INTEGRABLE SYSTEMS, NONASSOCIATIVE ALGEBRAS AND AFFINELY CONNECTED SPACES

S.I. Svinolupov)

The existence of effective integrability conditions for multicomponent evolution PDEs enables one to solve the problem of the classification of such equations. Investigations already done show that for polynomial systems of an arbitrary high number of equations classification results can be formulated in a natural way in terms of nonassociative algebraic structures. The theory of nonassociative algebras can be very helpful for the investigation of polynomial Miura and Backlund transformations and the classification of differential-difference systems (chains). Using methods of nonassociative algebras large classes of multicomponent polynomial integrable evolution equations of the second (Schrodinger type) and third (KdV type) orders are obtained. The integrable differential-difference systems generalizing the well-known Volterra lattice are constructed.

Nonpolynomial multicomponent equations can be described in terms of differential geometry (connectedness, curvature and torsion tensors, etc.). The correspondence between large classes multifield integrable systems and affinely connected spaces is established. The multicomponent integrable evolution equations of the Heisenberg type and differential-difference systems generalizing the Toda lattice are constructed. It is shown that each equation corresponds to affinely connected space specified by deformation of some nonassociative algebra.

# On the integrability of Henon-Heiles type systems

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We consider multidimensional extensions of the Henon-Heiles system obtained as stationary flows from the KdV hierarchy. These systems are shown to have a bi-Hamiltonian structure in a phase space extended in a suitable way. However this bi-Hamiltonian structure cannot be geometrically reduced onto the original phase space together with the associated Hamiltonian vector fields. For this reason, we propose a new integrability criterion holding for a generic finite-dimensional Hamiltonian system. Though weaker than the bi-Hamiltonian scheme, this criterion will be shown to assure Liouville-integrability of a Hamiltonian system in its standard phase space, i.e. without the introduction of supplementary coordinates. As an example we apply this criterion to a Henon-Heiles system with four degrees of freedom.

# Forced Lattice Vibrations

Stephanos Venakides

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We consider a nonlinear, semi-infinite particle chain with nearest neighbor interactions forced by an imposed constant velocity on the leading particle that tends to compress the chain. An oscillatory disturbance arises at the leading particle and travels along the chain leaving behind either a state of equilibrium or a state of binary oscillation, according to whether the driving particle is given a speed that is below or above a critical value. In the case of the integrable Toda chain, we analyze the critical behavior and we calculate the residual oscillatory supercritical structure in great detail. Our calculation explains phenomena observed numerically by von Neumann and by Holian and Straub. The regime of the calculation is strongly nonlinear. The calculation relies on the integrability of the Toda chain and employs an extension of the machinery of the Lax-Levermore theory that we have developed.

When the velocity driving the leading particle is periodic, the residual state of the chain can be either one of equilibrium or one dominated by single or multiphase waves. When the driving amplitude is small, we calculate a sequence of frequency thresholds to the penetration of the chain by single and multiphase waves. We construct the multiphase waves (in the integrable chain) that develop as well as the boundary layer that appears close to the driver. We can, at this stage, do the same only for single phase waves in the nonintegrable chain.

When the amplitude of the driver is not small and the chain is integrable we derive and analyze the eigenvalue dynamics that determine the residual state of the chain.



NEW GEOMETRICAL METHODS OF ANALYSIS OF NONLINEAR PDE

A. VINOGRADOV

The talk aims to exhibit informally Secondary Quantized Calculus, to present a menu of its applications to various nonlinear problems and to discuss some perspectives.

## THERMODYNAMIC FUNCTIONS OF NONLINEAR 1-D SYSTEMS

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The aim of this paper is to present a technique to obtain the equilibrium thermodynamic properties of nonlinear kink-bearing scalar fields. Using known asymptotic methods from the theory of differential equations depending on a large parameter, one calculates temperature and lattice corrections to the free energy. The method was used to calculate the multi-kink contribution to the free energy, as well as the three soliton contribution to the specific heat of a classical easy-plane ferromagnet chain.

The soliton is considered to have the properties of a classical particle, with a considerable stability against perturbations. This is a pure analytical method. The free energy will be calculated in the displacive (continuum) limit, and at low temperatures.

From the phenomenological point of view, the elementary excitations are the renormalized kinks and the low amplitude extended states (phonons). Obviously, that ideal gas phenomenology represents only an approximation of the real situation, but at low temperatures it is in perfect agreement with the exact results of the transfer integral operators.

In summary, the method consists in the following steps:

- The partition function, calculated using the transfer integral operator (TIO) method is determined in the thermodynamic limit by the lowest eigenvalue of TIO.
- In the continuum limit these eigenvalues can be found solving a Schrödinger-type equation, and at low temperatures they can be written as asymptotic series expansions in a small parameter.
- To solve the Schrödinger-type equation with the potential  $V(\phi)$  and with a large effective mass, one follows two steps:
  - based on Langer's transformation, one looks for an uniform valid expansion of the solution near the minimum of  $V(\phi)$ ;
  - in the second step, the existence of the other minima are taken into account using special symmetry properties of the wavefunction in special symmetry points of the potential.

# Symmetry Constraints of the KP hierarchy and a nonlocal Boussinesq equation.

by Ralph Willox

## Abstract:

A recently constructed sech-squared soliton system (the so called nonlocal Boussinesq or nlBq-equation) is known to be linked to Kaup's water wave-equation. It represents an interesting resonance-free alternative to the nonlinear string equation, but does not however bilinearize in any straightforward way.

A hereditary recursion operator can be constructed for this new equation, thus defining a hierarchy of commuting flows.

The nlBq-hierarchy defined in this manner, exhibits a very close link to the Kadomtsev-Petviashvili (KP) hierarchy, when one eventually manages to express it in terms of Hirota bilinear operators.

This link can be discussed in terms of a symmetry constraint on the KP spectral problem, hence clarifying a number of close relations which exist between the nlBq-equation and several other integrable systems. In particular, it will be shown that this constraint can be related to a symmetry constraint which reduces the KP-hierarchy to the Nonlinear Schrödinger hierarchy, as well as to a constraint on the modified KP-hierarchy reducing it in its turn to the Classical Boussinesq-hierarchy.

# SYMMETRY CLASSIFICATION OF DYNAMICAL SYSTEMS ON LATTICES.

Pavel WINTERNITZ

A method is presented for calculating the continuous Lie point symmetries of differential-difference equations and for classifying such systems into equivalence classes. The method is applied to equations of the form  $u(n,t)'' = F(t, u(n-1,t), u(n,t), u(n+1,t))$ , describing a system on a lattice with nearest neighbor interactions. The results were obtained in collaboration with D. Levi.

## LOCAL MASTER SYMMETRIES OF NONLINEAR INTEGRABLE EVOLUTION EQUATIONS

R.I. Yamilov (with I.Yu. Cherdantsev and S.I. Svinolupov)

As is known, master symmetries of nonlinear integrable equations are of great interest not only from the view point of the investigation of algebraic properties of integrable equations but also as examples of nonlinear equations explicitly depending on the spatial variables and the time and integrable in a special sense. We consider local evolution master symmetries similar to the master symmetry  $u_t = x(u_{xx} + 2uu_x) + u^2$  of the Burgers equation (compare to one of the KdV equation:  $u_t = x(u_{xxx} + 6uu_x) + 4(u_{xx} + 2u^2) + 2u_x \partial_x^{-1}(u)$ ). Such master symmetries generate not only higher symmetries of corresponding integrable equations but also conserved quantities and additional Hamiltonian structures.

We demonstrate that there are many instances in which integrable equations (both partial differential and differential-difference) possess local evolution master symmetries. Moreover, we present a modification of the symmetry approach which allows one to find and classify integrable equations with master symmetries of this kind.